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Optimal emitter-collector gap for thermionic energy converters

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In this letter, we calculate numerically the emitter-collector gaps that maximize the power conversion efficiency of vacuum thermionic energy converters (TECs). The optimum arises because efficiency drops both at very large gaps, due to space-charge limitations on the TEC current, and at very small gaps, due to the increased parasitic heat loss via near-field radiative heat transfer. For typical TECs made with cesiated tungsten electrodes, the optimal gaps range from 900 nm to 3 μm and are approximately equal to the characteristic wavelength of the emitter thermal radiation, as given by Wien’s displacement law. © 2012 American Institute of Physics.

Thermionic energy converters (TECs) are heat engines that convert heat directly to electricity at very high temperatures, typically >1000 °C. This energy conversion process is based on thermionic emission—the evaporation of electrons from conductors at high temperatures—and was first proposed in 1915. In the 1950s–1960s, the first practical TECs with conversion efficiencies of 10%–15% were created, and in the subsequent two decades, both NASA and the Soviet space programs developed TECs as power sources for space missions. These devices were fabricated using vacuum-tube technology and had electrode gaps on the order of 100 μm, formed by precision machining.

The energy diagram of a TEC is shown in Fig. 1. As the emitter is heated to high temperatures, the thermal distribution develops a long high-energy tail so that some electrons begin to overcome the work function barrier and evaporate from the hot emitter. The electrons can then cross the vacuum inter-electrode gap, condense at the relatively cool collector, and finally return to the emitter through an external load using the difference between the Fermi levels of the two electrodes to perform useful work. The thermionic currents, emitted from both emitter and collector, are governed by the Richardson-Dushman law.

Space charge between the electrodes can dramatically reduce the output power and efficiency of TECs because the electrons traversing the inter-electrode gap repel each other. For macroscopic gaps (>100 μm), the resulting additional energy barrier (Fig. 1) can reduce the output power and the conversion efficiency by many orders of magnitude. As a result, early TECs in the 1950s–1980s used ignited cesium plasma to neutralize space charge between the electrodes. Such plasma TECs achieved high output powers, but only at the cost of greatly increased complexity and decreased maximum efficiency.

As an alternative to using plasma, the deleterious effects of space charge can also be mitigated by making the inter-electrode gap small enough so that there is not enough space to develop a significant additional energy barrier. It has long been known that such vacuum TECs can be more efficient than plasma TECs if micron-scale gaps are used (<10 μm). This fact has been a motivation for recent efforts to develop microfabricated thermionic energy converters.

However, as we demonstrate in this letter, the gap can also be too small. At gaps of a few microns or less, the distance between the electrodes becomes smaller than the characteristic wavelength of thermal radiation, given by Wien’s displacement law. As a result, the traditional Stefan-Boltzmann formula, valid only for far-field propagating-wave radiative heat transfer, can no longer be used. Instead, near-field evanescent-wave radiative heat transfer, which is sometimes referred to as photon tunneling, starts to dominate. The total radiative heat transfer can then be increased by many orders of magnitude, as recently demonstrated both theoretically and experimentally.

![FIG. 1. The energy diagram of the thermionic energy conversion process.](http://dx.doi.org/10.1063/1.4707379)
For given temperatures of the emitter and the collector, this increase in radiative heat transfer does not affect the output power of the TEC. However, it increases the necessary heat input to the emitter and therefore reduces its power conversion efficiency. The interplay between the effects of space charge and near-field radiative heat transfer means that there is an optimal gap (or a range of gaps) on the order of a few microns, which maximizes the energy conversion efficiency. We note that at much smaller distances, on the order of a few nanometers, electrons begin to tunnel across the gap, potentially resulting in another peak in efficiency. How-

er, in this letter, we focus on the currently experimentally feasible⁹ micron-range gaps. In particular, we calculate the optimal conditions for electrodes made of cesiated tungsten, which have traditionally been used in TECs.²⁻⁴

The energy conversion efficiency of a TEC can be written as follows:

\[ \eta = \frac{P_{out}}{P_{in}} = \frac{J_{net}(V_0 - J_{net}R_{lead})}{Q_{total}}, \]

where \( V_0 \) is the voltage difference between two electrodes, \( J_{net} \) is the net electron current density, \( R_{lead} \) is the total resistance of the leads between the electrodes and the external load (Fig. 1), normalized to unit area, and \( Q_{total} \) is the total energy flux between the emitter and the collector.

According to the Richardson-Dushman equation, the net current, \( J_{net} \), is the difference between the thermionic currents from the emitter and the collector

\[ J_{net} = J_E - J_C = AT_E^2 \exp\left(-\frac{\phi_E + \phi_{ME}}{k_BT_E}\right) - AT_C^2 \exp\left(-\frac{\phi_C + \phi_{MC}}{k_BT_C}\right), \]

where \( J_E \) and \( J_C \) are the thermionic current densities from emitter and collector, respectively, and \( A \) is a material-specific Richardson-Dushman constant, which we assume equal to the ideal value of 1.2 \( \times \) 10⁶ Am⁻²K⁻² for simplicity. \( T_E \) and \( T_C \) are the temperatures of the emitter and the collector, respectively. The additional barriers, \( \phi_{ME} \) and \( \phi_{MC} \), are due to the electrostatic field created by the electrons between the electrodes and can be calculated using Langmuir’s space-charge theory,¹³ as described in detail in Sec. 10.3 of Ref. 3. The output voltage is then given by \( V_0 = (\phi_E + \phi_{ME}) - (\phi_C + \phi_{MC}). \)

The thermionic current originating from the collector, \( J_C \), corresponds to electrons flowing in the wrong direction, i.e., from the collector to emitter. The collector current becomes negligible (\( J_C << J_E \)) if the collector temperature is small enough; however, for some regimes of operation, the reverse current from the collector can become significant mechanism of energy loss, reducing the output power and conversion efficiency.

The net energy input into the emitter is equal to the total energy flux from emitter to collector, \( Q_{total} = Q_{thermionic} + Q_{prop} + Q_{evan} + Q_{lead} \), due to a number of fundamental energy carriers: thermionically emitted electrons \( (Q_{thermionic}) \), propagating-wave photons \( (Q_{prop}) \), evanescent-wave photons \( (Q_{evan}) \), and electron heat conduction in the leads \( (Q_{lead}) \).

The net energy carried by thermionically emitted electrons from emitter to collector is given by²

\[ Q_{thermionic} = \frac{\varphi_E + \varphi_{ME}}{e} J_{net} + \frac{2k_B}{e} (J_E T_E - J_C T_C). \]

A part of this thermionic energy flux, namely, \( J_{net}V_0 \), is converted to electricity with the rest deposited in the collector as heat when thermionic electrons are absorbed into the collector. The propagating-wave heat transfer is described by the familiar Stefan–Boltzmann law at large inter-electrode gaps. However, when two surfaces approach close enough such that the gap is on the order of the characteristic wavelength of thermal radiation given by Wien’s displacement law, \( \lambda_{ch} \approx 2.9 \times 10^{-3} m \cdot K/T_E \), the propagating-wave heat flux gets modified, but even more importantly, the evanescent waves from each surface begin to couple to each other and exchange energy. If the optical properties of both electrodes are known, the two components of the radiation heat flux can be calculated as follows:¹⁰

\[ Q_{prop} = \frac{1}{\pi^2} \int_{0}^{\infty} \omega \left[ \theta(\omega, T_E) - \theta(\omega, T_C) \right] \cdot \int_{0}^{\pi} s_{prop}(\omega, \beta, \epsilon_1(\omega), \epsilon_2(\omega)) d\beta, \]

\[ Q_{evan} = \frac{1}{\pi^2} \int_{0}^{\infty} \omega \left[ \theta(\omega, T_E) - \theta(\omega, T_C) \right] \cdot \int_{0}^{\infty} s_{evan}(\omega, \beta, \epsilon_1(\omega), \epsilon_2(\omega)) d\beta, \]

where \( \theta(\omega, T) \) is the mean energy of a Planck oscillator at the angular frequency \( \omega \), \( \beta \) is the parallel wavevector, which must be equal for both surfaces, \( \epsilon_1(\omega) \) and \( \epsilon_2(\omega) \) are the relative permittivities of the emitter and the collector, respectively, and \( s_{prop} \) and \( s_{evan} \) are the coupling coefficients for propagating and evanescent waves, which are defined in Ref. 10.

Historically, many TECs have been made of tungsten, whose optical properties in the infrared range, where most radiative heat transfer occurs, are described reasonably well by the Drude model¹⁴

\[ \epsilon = 1 - \frac{\sigma_0/\tau}{\omega_0 (\omega^2 + i\omega/\tau)}, \]

where \( \sigma_0 \) is dc conductivity and \( \tau \) is electron relaxation time. Since the dc conductivity depends on temperature, we used a 6th order polynomial to fit the published data of tungsten’s resistivity versus temperature.¹⁵ The electron relaxation time was estimated as \( \tau = 1/(2T^2 + \beta T^3) \) with \( \alpha = 10^7 \text{s}^{-1} \cdot \text{K}^{-2} \) and \( \beta = 2 \times 10^6 \text{s}^{-1} \cdot \text{K}^{-3} \) (Ref. 14).

Since TECs are high-current devices, the electrical resistance of the leads that connect the electrodes to the external load should be optimized.² If the electrical resistance of the lead is too large, most of the potentially available output power will be wasted on Joule heating in the leads. Conversely, if the electrical resistance of the lead is too small, the heat conduction through the leads will be large, and

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maintaining the temperature of the emitter will require excessive input power. This is due to the Wiedemann–Franz law, which relates the minimum heat conduction through metal leads to the electrical resistance of the leads

\[ Q_{\text{Lead}} = \frac{L}{2R_{\text{lead}}} (T_E^2 - T_C^2), \]

where the Lorenz number \( L = 2.24 \times 10^{-8} \text{W} \cdot \text{K}^{-2} \). The optimal lead resistance that maximizes the conversion efficiency can be easily calculated analytically if all other parameters are known.\(^2\)

Using the formulas above, we numerically calculated the \( I-V \) curves, the maximum output power, and the other energy fluxes as a function of the interelectrode gap for any given values of \( T_E, T_C, \phi_E \), and \( \phi_C \). FIG. 2 shows one example, where \( T_E = 2000\text{K}, T_C = 900\text{K}, \phi_E = 2.67\text{eV}, \) and \( \phi_C = 1.5\text{eV} \). We chose \( \phi_C = 1.5\text{eV} \) because it is approximately the lowest work function obtainable for cesiated tungsten and \( \phi_E = 2000/750 = 2.67\text{eV} \) because it is approximately optimal for this emitter temperature.\(^2\)

For gaps above a few microns, space charge becomes significant, and the output power falls off rapidly. Conversely, for gaps below a micron, the total radiative heat transfer increases rapidly due to the evanescent-wave heat transfer. The thermionic energy conversion process is generally most efficient when the total energy flux is dominated by the thermionic component, \( Q_{\text{thermionic}} \), so that the ratio of output power to total energy flux is roughly constant.

The energy conversion efficiency can also be plotted versus gap for any given values of \( T_E, T_C, \phi_E, \) and \( \phi_C \), as illustrated in FIG. 3. As expected, the efficiency is small at both very large gaps and very small gaps and reaches the maximum at gaps on the order of a micron. In FIG. 3, we also defined the optimal gap range, where the energy conversion efficiency is at least 0.9 of the maximum. The optimal gaps generally range from a few tenths of a micron to a few microns. Similar graphs were then generated by varying the emitter temperature from 400 K to 2600 K and assuming the emitter work function of \( T_E[K]/750\text{eV} \), which is approximately optimal for vacuum TECs (Sec. 2.8 of Ref. 2). In addition, the collector work function was varied from 0.5 to 1.5 eV to compare the efficiencies of tungsten micron-scale thermionic converters that could potentially be achieved in the future with more effective work-function-lowering coatings. For all work functions, we assumed that the collector temperature is \( \phi_C[\text{eV}] \times 600\text{K} \), again because it is approximately optimal for TECs (Sec. 2.8 of Ref. 2). Such work functions are feasible for cesiated tungsten electrodes if the liquid cesium reservoir is maintained at a temperature of 600–700 K (Sec. 4.2 of Ref. 2).

FIG. 2. The total energy flux, the heat transfer through propagating and evanescent waves, the maximum output power per unit area (corresponding to \( R_{\text{lead}} = 0 \)), and the heat transferred by thermionically emitted electrons (for the same maximum power bias conditions) as a function of interelectrode gap for \( T_E = 2000\text{K}, T_C = 900\text{K}, \phi_E = 2000/750 = 2.67\text{eV}, \) and \( \phi_C = 900/600 = 1.5\text{eV} \). The inset shows \( I-V \) curves for several gaps. The circles signify the bias conditions corresponding to maximum output power. At large gaps, propagating-wave heat flux is approximately constant and can be approximated by the Stefan-Boltzmann formula, \( Q_{\text{prop}} \approx \epsilon_{\text{em}} (T_E^4 - T_C^4) \) with the effective emissivity \( \epsilon_{\text{em}} \approx 0.07 \).

FIG. 3. Maximum energy conversion efficiency versus gaps for emitter temperatures of 1500, 2000, and 2500 K, collector temperature of 900 K, and collector work function of 1.5 eV. The emitter work functions were assumed to be \( T_E[K]/750\text{eV} \). The optimal gap ranges were from 300 nm to 6 \( \mu \)m for \( T_E = 1500\text{K} \), from 200 nm to 4 \( \mu \)m for \( T_E = 2000\text{K} \), and from 150 nm to 3 \( \mu \)m for \( T_E = 2500\text{K} \).
face waves. However, the optimal gaps are still likely to depend on the optical properties of the electrodes. In fact, some materials manifest additional evanescent heat transfer due to resonant surface waves.

However, the optimal gaps are still likely to depend on the optical properties of the electrodes. In fact, some materials manifest additional evanescent heat transfer due to resonant surface waves. However, the optimal gaps are still likely to be on the order of a few microns for all materials because space charge generally becomes significant at gaps above 10 μm, and all mechanisms of near-field radiative heat transfer begins to dominate at distances of a few microns or less at the temperatures of interest to TECs. The significance of these results is that the optimal gaps of vacuum TECs can be conveniently achieved using MEMS fabrication technology. Moreover, at least for tungsten electrodes, microfabricated TECs have a relatively wide range of optimal gaps (more than an order of magnitude), indicating that they should be robust to changes in the gap during operation.

In summary, we have shown that due to the interplay between space charge and near-field radiative heat transfer, there is an optimal inter-electrode gap that maximizes the energy conversion efficiency of vacuum TECs. The efficiency is typically maximized for gaps similar to the characteristic wavelength of thermal radiation of the emitter. For the particular case of cesiated tungsten electrodes, the efficiency is nearly optimal for gaps between 900 nm and 3 μm over a wide range of operating conditions. Microfabricated vacuum TECs therefore show great promise for achieving efficient conversion of heat directly to electricity and have potential application in topping cycles for concentrated solar thermal power plants.

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